

# A note on the hidden conformal structure of non-extremal black holes

Tomás Ortín <sup>a</sup> and C. S. Shahbazi <sup>b</sup>

*Instituto de Física Teórica UAM/CSIC  
C/ Nicolás Cabrera, 13–15, C.U. Cantoblanco, 28049 Madrid, Spain*

## Abstract

We study, following Bertini *et al.* [1], the hidden conformal symmetry of the massless Klein-Gordon equation in the background of the general, charged, spherically symmetric, static black-hole solution of a class of  $d$ -dimensional Lagrangians which includes the relevant parts of the bosonic Lagrangian of any ungauged supergravity. We find that a hidden  $SL(2, \mathbb{R})$  symmetry appears at the near event- and Cauchy-horizon limits. We extend the two  $\mathfrak{sl}(2)$  algebras to two full Witt algebras (Virasoro algebras with vanishing central charges). We comment on the implications of the possible existence of an associated quantum conformal field theory.

---

<sup>a</sup>E-mail: Tomas.Ortin [at] csic.es

<sup>b</sup>E-mail: Carlos.Shabazi [at] uam.es

# Introduction

A complete microscopic explanation of the entropy of an arbitrary black hole remains as an outstanding challenge for Theoretical Physics. In the mid 90's, the microscopic degrees of freedom of a charged, static, extremal, black hole in 5 non-compact dimensions were explicitly identified in the framework of String Theory, in complete agreement with the Bekenstein-Hawking entropy [2], providing a first breakthrough in this quest. The microscopical entropy of many other 4- and 5-dimensional black holes has been computed successfully following the same pattern.

Although these results were initially thought to depend on the specific features of String Theory, it has become clear that this is not the case and the UV details are not important in order to just understand the area law from a microscopic point of view. The existence of a UV completion, although important from a fundamental point of view, seems to be irrelevant for this purpose. This irrelevance strongly suggests the existence of an universal underlying principle, which is included in, but not exclusive of String Theory, which justifies these calculations<sup>1</sup>.

A major step in this direction was taken in [4] with the study of the  $(2+1)$ -dimensional BTZ black hole [5]: it had been already shown in [6] that the asymptotic symmetry algebra of this solution was a Virasoro algebra; therefore any consistent quantum theory describing this black hole should be a conformal field theory, and hence the Cardy formula can be used to compute the asymptotical growth of states, obtaining a result that is in agreement with the Bekenstein-Hawking entropy. This analysis (with some differences, such as considering the symmetries in the near horizon limit) has been extended to other, higher-dimensional black holes [7], and a seemingly universal characteristic of all the black holes whose entropy has been computed microscopically has emerged: they all are described by 2-dimensional conformal theories, at least in some limit.

A considerable effort has been dedicated to unveil the hidden conformal symmetries of the near-horizon region of different kinds of black holes. For instance, in [8], a duality between the extremal Kerr black hole and a chiral 2-dimensional conformal theory was found. For the non-extremal Kerr black hole, a different approach has been adopted in [9], where the massless Klein-Gordon equation was used in order to elucidate the hidden conformal symmetry. In particular, it was shown that it is possible to define a set of vector fields of a particular submanifold of the space-time, such as they obey the  $\mathfrak{sl}(2)$  algebra and the Casimir gives the massless wave operator. This approach has later been used in [1] and [10] (see also [20]) for the Schwarzschild and the Kerr-Newman black holes, respectively<sup>2</sup>, and it is the one that we are going to use for general  $d$ -dimensional black holes in this note, using the metrics introduced in [12] and [13]<sup>3</sup>.

The note is organized as follows: in Section 1 we present the theories that we consider and the generic black-hole metrics that we will use as a background for the massless Klein-Gordon equation. In Section 2 we will study of the hidden conformal symmetry in the near-horizon regions (inner and outer) of the 4-dimensional case. The  $d$ -dimensional generalization is made in the next Section and we discuss our results in Section 4.

---

<sup>1</sup>For a recent and comprehensive review of these ideas see [3].

<sup>2</sup>Previous, closely related results were published in [11].

<sup>3</sup>The search for the hidden conformal symmetry in static black holes has a long history. See, for example, [14, 15, 16], and, more recently, [17].

# 1 The background metric

We are going to consider black-hole solutions of 4-dimensional theories of the general form

$$I = \int d^4x \sqrt{|g|} \left\{ R + \mathcal{G}_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + 2\Im \mathcal{N}_{\Lambda\Sigma} F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} - 2\Re \mathcal{N}_{\Lambda\Sigma} F^\Lambda_{\mu\nu} \star F^{\Sigma\mu\nu} \right\}, \quad (1.1)$$

which includes the bosonic sectors of all 4-dimensional ungauged supergravities for appropriate  $\sigma$ -model metrics  $\mathcal{G}_{ij}(\phi)$  and kinetic matrices  $\mathcal{N}_{\Lambda\Sigma}(\phi)$  with negative-definite imaginary part. The indices  $i, j, \dots$  run over the scalar fields and the indices  $\Lambda, \Sigma, \dots$  over the 1-form fields. Their numbers are related only for  $N \geq 2$  supergravity theories.

The metrics of all spherically symmetric, static, black-hole solutions of the action Eq. (1.1) have the general form [12]

$$ds^2 = e^{2U} dt^2 - e^{-2U} \gamma_{mn} dx^m dx^n, \quad (1.2)$$

$$\gamma_{mn} dx^m dx^n = \left( \frac{r_0}{\sinh r_0 \tau} \right)^2 \left[ \left( \frac{r_0}{\sinh r_0 \tau} \right)^2 d\tau^2 + d\Omega_{(2)}^2 \right],$$

where  $r_0$  is the non-extremality parameter and  $U(\tau)$  is a function of the radial coordinate  $\tau$  that characterizes each particular solution. In these coordinates the exterior of the event horizon is covered by the negative values of  $\tau$ , the event horizon being located at  $\tau \rightarrow -\infty$  and spatial infinity at  $\tau \rightarrow 0^-$ . The interior of the Cauchy horizon (if any) is covered by part of the positive values of  $\tau$ , the inner horizon being located at  $\tau \rightarrow +\infty$  while the singularity is located at some finite, positive, value of  $\tau$  [18].

The last term in the action Eq. (1.1) can only occur in  $d = 4$  dimensions. Therefore, in the general  $d$ -dimensional case we shall consider the Lagrangian

$$I = \int d^d x \sqrt{|g|} \left\{ R + \mathcal{G}_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + 2I_{\Lambda\Sigma}(\phi) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} \right\}, \quad (1.3)$$

where  $I_{\Lambda\Sigma}(\phi)$  is an invertible, negative-definite, scalar-dependent matrix. The metrics of the spherically symmetric, static, black-hole solutions of (1.3) have the general form [13]

$$ds^2 = e^{2U} dt^2 - e^{-\frac{2}{d-3}U} \gamma_{mn} dx^m dx^n, \quad (1.4)$$

$$\gamma_{mn} dx^m dx^n = \left( \frac{\mathcal{B}}{\sinh \mathcal{B}\rho} \right)^{\frac{2}{d-3}} \left[ \left( \frac{\mathcal{B}}{\sinh \mathcal{B}\rho} \right)^2 \frac{d\rho^2}{(d-3)^2} + d\Omega_{(d-2)}^2 \right].$$

Here  $\mathcal{B}$  is the higher-dimensional generalization of the non-extremality parameter  $r_0$  and the metric is well defined and covers the exterior of the event horizon for positive values of  $\rho$ , the event horizon being at  $\rho \rightarrow +\infty$  and spatial infinity at  $\rho \rightarrow 0^+$ .

If the above metric describes the exterior of a regular black hole, one can find from it the metric that covers the interior of the Cauchy horizon (if any) that metric according to [19]

$$\rho \longrightarrow -\varrho, \quad e^{-U^{(+)}(\rho)} \equiv e^{-U(\rho)} \longrightarrow -e^{-U(-\varrho)} \equiv -e^{-U^{(-)}(\varrho)}. \quad (1.5)$$

The new metric, determined by the function  $U^{(-)}$  has the same general form in terms of the coordinate  $\varrho$  which now takes values in the range  $\varrho \in (\varrho_{\text{sing}}, +\infty)$  because the metric will generically hit a singularity before  $\varrho$  reaches 0: if the original  $e^{-U^{(+)}}$  is always finite for positive values of  $\rho$ , the transformed one will have a zero for some finite positive value of  $\varrho$ .

In the 4-dimensional case, the area of a 2-sphere at fixed radial coordinate  $\tau = \tau_0$  is given by

$$A(\tau_0) = 4\pi f^2(\tau_0) e^{-2U(\tau_0)}, \quad (1.6)$$

where

$$f(\tau) \equiv \frac{r_0}{\sinh r_0 \tau}. \quad (1.7)$$

Therefore, the areas of the event and Cauchy horizons,  $A_+$  and  $A_-$ , respectively, are given by

$$A_{\pm} = \lim_{\tau_0 \rightarrow \mp \infty} A(\tau_0). \quad (1.8)$$

In the  $d$ -dimensional case, we can write a common expression for the area of a  $(d-2)$ -sphere at fixed radial coordinate  $\rho = \rho_0 > 0$  in the exterior of the event horizon or  $\varrho = \rho_0 > 0$  in the interior of the Cauchy horizon:

$$A(\rho_0) = C_{d-2} \left| e^{-U^{(+)}(\rho_0)} g(\rho_0) \right|^{\frac{d-2}{d-3}}, \quad (1.9)$$

where

$$C_{(d-2)} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)}, \quad (1.10)$$

is the volume of the round  $(d-2)$ -sphere of unit radius and

$$g(\rho) \equiv \frac{\mathcal{B}}{\sinh \mathcal{B}\rho}. \quad (1.11)$$

The area of the outer (+) and inner (−) horizons,  $A_{\pm}$  are given by

$$A_{\pm} = \lim_{\rho_0 \rightarrow \pm \infty} A(\rho_0), \quad (1.12)$$

We will use Eqs. (1.8) and (1.12) later in order to interpret the near-horizon limits of the massless Klein-Gordon equation.

## 2 The massless Klein-Gordon equation in a general static black hole background

In [1] it was shown that the massless Klein-Gordon equation in the background of a 4-dimensional black hole exhibits a  $SL(2, \mathbb{R})$  invariance in the near-horizon limit which extends to spatial infinity at sufficiently low frequencies. Here we will generalize these results to the charged, static,

spherically symmetric black-hole solutions of 4-dimensional theories of the form Eq. (1.1), with metrics of the general form Eq. (1.2).

In the space-time background given by the metric (1.2), the massless Klein-Gordon equation

$$\frac{1}{\sqrt{|g|}} \partial_\mu \left( \sqrt{|g|} g^{\mu\nu} \partial_\nu \Phi \right) = 0, \quad (2.1)$$

can be written in the form

$$e^{-2U} \partial_t^2 \Phi - e^{2U} f^{-4} \partial_\tau^2 \Phi - e^{2U} f^{-2} \Delta_{S^2} \Phi = 0, \quad (2.2)$$

where  $f(\tau)$  has been defined in Eq. (1.7) and

$$\Delta_{S^2} \Phi = \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta \Phi) + \frac{1}{\sin^2 \theta} \partial_\phi^2 \Phi, \quad (2.3)$$

is the Laplacian on the round 2-sphere of unit radius. Using the separation ansatz

$$\Phi = e^{-i\omega t} R(\tau) Y_m^l(\theta, \phi), \quad (2.4)$$

and

$$\Delta_{S^2} Y_m^l(\theta, \phi) = -l(l+1) Y_m^l(\theta, \phi), \quad (2.5)$$

we find

$$\omega^2 e^{-4U} f^2 R(\tau) + f^{-2} \partial_\tau^2 R(\tau) = l(l+1) R(\tau), \quad (2.6)$$

so we can write Eq. (2.2) as

$$\mathcal{K}_4 \Phi = l(l+1) \Phi, \quad (2.7)$$

where  $\mathcal{K}_4$  is the second-order differential operator

$$\mathcal{K}_4 \equiv -e^{-4U} f^2 \partial_t^2 + f^{-2} \partial_\tau^2. \quad (2.8)$$

In order to exhibit the hidden conformal structure of the given space-time, we want to find a representation of  $SL(2, \mathbb{R})$  in terms of first-order differential operators (vector fields) in the  $t - \tau$  submanifolds, such as the  $SL(2, \mathbb{R})$  quadratic Casimir, constructed from those vector fields is equal to the second-order differential operator  $\mathcal{K}_4$ . Thus, we want to find three real vector fields

$$L_m = a_{mt} \partial_t + a_{m\tau} \partial_\tau, \quad m = 0, \pm 1, \quad (2.9)$$

for some functions  $a_{mt}(t, \tau), a_{m\tau}(t, \tau)$ , whose Lie brackets are satisfy  $\mathfrak{sl}(2)$  Lie algebra

$$[L_m, L_n] = (m - n) L_{m+n}, \quad m = 0, \pm 1, \quad (2.10)$$

and such that

$$\mathcal{H}^2 \equiv L_0^2 - \frac{1}{2} (L_1 L_{-1} + L_{-1} L_1) = \mathcal{K}_4. \quad (2.11)$$

In order to simplify this problem, following [1], we have to make some additional assumptions on the functions  $a_{It}(t, \tau)$ ,  $a_{I\tau}(t, \tau)$ . Thus, we make the following ansatz

$$L_1 = l(t) [-m(\tau)\partial_t + n(\tau)\partial_\tau] , \quad (2.12)$$

$$L_0 = -\frac{c}{r_0}\partial_t , \quad (2.13)$$

$$L_{-1} = -l^{-1}(t) [m(\tau)\partial_t + n(\tau)\partial_\tau] , \quad (2.14)$$

where  $m$  and  $n$  are functions of  $\tau$ ,  $l$  is a function of  $t$  and  $c$  is a real constant.

Plugging this ansatz into Eq. (2.10) we obtain two differential equations

$$m^2\partial_t \log l + n\partial_\tau m = \frac{c}{r_0} , \quad (2.15)$$

$$\frac{c}{r_0}\partial_t \log l = 1 , \quad (2.16)$$

and plugging it into Eq. (2.11) we obtain three equations

$$m = h\partial_\tau n , \quad (2.17)$$

$$m^2 = e^{-4U}f^2 + (c/r_0)^2 , \quad (2.18)$$

$$n^2 = f^{-2} . \quad (2.19)$$

These equations cannot be solved for arbitrary  $U(\tau)$ : we can find  $l, m, n$  as functions of  $f(\tau)$  and the constant  $c$

$$l(t) = c_0 e^{r_0 t/c} , \quad n^2(\tau) = f^{-2} , \quad m(\tau) = h \cosh(r_0 \tau) , \quad (2.20)$$

for some real constant  $c_0$ , leaving the following equation for the constant  $c$  to be solved:

$$c^2 = (e^{-2U}f^2)^2 . \quad (2.21)$$

This equation can only be exactly solved, for all values of the radial coordinate  $\tau$  for  $e^U \sim f$ , which does not correspond to any asymptotically flat black hole. We have to content ourselves with a range of values of the coordinate  $\tau$  in which the above equation can be solved approximately. The two ranges that we have identified correspond to the two near-horizon regions (event and Cauchy horizons  $\tau \rightarrow -\infty$  or  $\tau \rightarrow +\infty$ , respectively) in which

$$(e^{-2U}f^2)^2 \stackrel{\tau \rightarrow \mp\infty}{\sim} \left(\frac{A_\pm}{4\pi}\right)^2 + \mathcal{O}(e^{\pm r_0 \tau}) = c^2 + \mathcal{O}(e^{\pm r_0 \tau}) , \quad (2.22)$$

according to Eq. (1.8).

We conclude that in the geometry of any 4-dimensional, charged, static, black-hole solution of a theory of the form Eq. (1.1), there are two triplets of vector fields  $L_m^+$  and  $L_m^-$ ,  $m = 0, \pm 1$  given by

$$L_1^\pm = -\frac{e^{r_0\pi t/S_\pm}}{r_0} \left( \frac{S_\pm}{\pi} \cosh(r_0\tau) \partial_t + \sinh(r_0\tau) \partial_\tau \right) \quad (2.23)$$

$$L_0^\pm = -\frac{S_\pm}{r_0\pi} \partial_t, \quad (2.24)$$

$$L_{-1}^\pm = -\frac{e^{-r_0\pi t/S_\pm}}{r_0} \left( \frac{S_\pm}{\pi} \cosh(r_0\tau) \partial_t - \sinh(r_0\tau) \partial_\tau \right), \quad (2.25)$$

where  $S_\pm = \frac{A_\pm}{4}$ , which generate two  $\mathfrak{sl}(2)$  algebras whose quadratic Casimirs

$$\mathcal{H}^{\pm 2} \equiv (L_0^\pm)^2 - \frac{1}{2} (L_1^\pm L_{-1}^\pm + L_{-1}^\pm L_1^\pm), \quad (2.26)$$

approximate the massless Klein-Gordon equation in the two near-horizon regions<sup>4</sup>:

$$\mathcal{K}_4 \Phi = \left\{ -e^{-4U} f^2 \partial_t^2 + f^{-2} \partial_\tau^2 \right\} \Phi \xrightarrow{\tau \rightarrow \mp \infty} f^{-2} \left\{ - (S_\pm/\pi)^2 \partial_t^2 + \partial_\tau^2 \right\} \Phi = \mathcal{H}^{\pm 2} \Phi. \quad (2.28)$$

We can see from Eq. (2.23) that the extremal limit  $r_0 \rightarrow 0$  is singular. The reason is that the operations of taking the near-horizon limit and of taking the extremal limit  $r_0 \rightarrow 0$  do not commute.

The  $\mathfrak{sl}(2)$  algebra that we have just found can be immediately extended to a complete Witt algebra (or a Virasoro algebra with vanishing central charge) with the commutation relations (2.10) for all  $m \in \mathbb{Z}$ . The generators of the Witt algebra are given by

$$L_m^\pm = -\frac{e^{mr_0\pi t/S_\pm}}{r_0} \left( \frac{S_\pm}{\pi} \cosh(mr_0\tau) \partial_t + \sinh(mr_0\tau) \partial_\tau \right). \quad (2.29)$$

### 3 Hidden conformal symmetry in $d$ dimensions.

We are now ready to generalize the results of the previous section to arbitrary  $d \geq 4$  dimensions, using the general metric Eq. (1.4). In this background, the massless Klein-Gordon equation can be written as

$$e^{-\frac{2(d-2)}{(d-3)}U} g^{\frac{2}{d-3}} \partial_t^2 \Phi - (d-3)^2 g^{-2} \partial_\rho^2 \Phi - \Delta_{S^{d-2}} \Phi = 0, \quad (3.1)$$

---

<sup>4</sup>Observe that we only approximate some terms (i.e. we keep some sub-dominating terms):

$$e^{-4U} f^2 = f^{-2} (e^{-2U} f^2)^2 \sim f^{-2} \left[ \left( \frac{A_\pm}{4\pi} \right)^2 + \mathcal{O}(e^{\pm r_0\tau}) \right] \sim f^{-2} \left( \frac{A_\pm}{4\pi} \right)^2 + \mathcal{O}(e^{\pm r_0\tau}), \quad (2.27)$$

which is correct to that order. On the other hand, we do not need to restrict ourselves to any particular range of frequencies.

where  $g(\rho)$  is defined in Eq. (1.11) and  $\Delta_{S^{d-2}}$  is the Laplacian in the round  $(d-2)$ -sphere of unit radius. Using the separation ansatz

$$\Phi = e^{-i\omega t} R(\rho) Y_\mu^l(\vec{\theta}), \quad (3.2)$$

where  $Y_\mu^l(\vec{\theta})$  are the spherical harmonics on  $S^{d-2}$ , Eq. (3.1) takes the form

$$\frac{e^{-\frac{2(d-2)}{(d-3)}U}}{(d-3)^2} g^{\frac{2}{d-3}} \omega^2 R(\rho) + g^{-2} \partial_\rho^2 R(\rho) = \frac{l(l+d-3)}{(d-3)^2} R(\rho), \quad (3.3)$$

so the Klein-Gordon equation takes the form

$$\mathcal{K}_d \Phi = \frac{l(l+d-3)}{(d-3)^2} \Phi. \quad (3.4)$$

where we have defined the reduced Klein-Gordon operator  $\mathcal{K}_d$

$$\mathcal{K}_d = -\frac{e^{-\frac{2(d-2)}{(d-3)}U}}{(d-3)^2} g^{\frac{2}{d-3}} \partial_t^2 + g^{-2} \partial_\rho^2. \quad (3.5)$$

As in the four-dimensional case, we want to find two triplets of vector fields generating the  $\mathfrak{sl}(2)$  Lie algebra and whose quadratic Casimir approximates the  $d$ -dimensional reduced Klein-Gordon operator  $\mathcal{K}_d$  in some region of the black-hole spacetime. It is not too hard to show that the two triplets

$$L_1^\pm = -\frac{e^{(d-3)C_{(d-2)}\mathcal{B}t/A_\pm}}{\mathcal{B}} \left( \frac{A_\pm}{(d-3)C_{(d-2)}} \cosh(\mathcal{B}\tau) \partial_t + \sinh(\mathcal{B}\tau) \partial_\tau \right) \quad (3.6)$$

$$L_0^\pm = -\frac{A_\pm}{(d-3)C_{(d-2)}\mathcal{B}} \partial_t, \quad (3.7)$$

$$L_{-1}^\pm = -\frac{e^{-(d-3)C_{(d-2)}\mathcal{B}t/A_\pm}}{\mathcal{B}} \left( \frac{A_\pm}{(d-3)C_{(d-2)}} \cosh(\mathcal{B}\tau) \partial_t - \sinh(\mathcal{B}\tau) \partial_\tau \right), \quad (3.8)$$

where Eqs. (1.9) and (1.12) have been used in order to take the near horizon  $\rho \rightarrow \pm\infty$  limit.

Extending these two  $\mathfrak{sl}(2)$  algebras to two full Witt algebras is straightforward:

$$L_m^\pm = -\frac{e^{m(d-3)C_{(d-2)}\mathcal{B}t/A_\pm}}{\mathcal{B}} \left( \frac{A_\pm}{(d-3)C_{(d-2)}} \cosh(m\mathcal{B}\tau) \partial_t + \sinh(m\mathcal{B}\tau) \partial_\tau \right). \quad (3.9)$$

## 4 Discussion

In this paper we have constructed two Witt algebras which have a well-defined action in the space of solutions to the wave equation in the background of the exterior and interior near-horizon limits of a generic, charged, static black hole. The two  $\mathfrak{sl}(2)$  subalgebras are symmetries



of these wave equations, since the wave operators can be seen as their Casimirs, but they are not symmetries of the background metrics which, being essentially the products of Rindler spacetime (locally Minkowski) and spheres, have Abelian (in the time-radial part) isometry algebras.

This result generalizes those obtained in [1, 10, 20, 11], and present an opportunity to put to test some conjectures and common lore of this field. To start with, is there a CFT associated to the Witt algebras and can one compute the central charge of the Virasoro algebra? A most naive computation does not seem to give meaningful results. This, of course, does not preclude the possibility that a more rigorous calculation, preceded of careful definitions of the boundary conditions of the fields at the relevant boundaries (which have to be identified first) may give a meaningful answer.

Meanwhile, it is amusing to speculate on the possible consequences of the existence of such a CFT with the left and right sectors whose entropies  $S_R, S_L$  and temperatures would be related to the temperatures and entropies of the outer and inner horizons ( $T_+, T_-$  and  $S_+, S_-$ , respectively) by

$$S_{\pm} = S_R \pm S_L, \quad (4.1)$$

$$\frac{1}{T_{\pm}} = \frac{1}{2} \left( \frac{1}{T_R} \pm \frac{1}{T_L} \right), \quad (4.2)$$

and obeying the fundamental relation

$$S_+ = \frac{\pi^2}{12} (c_R T_R + c_L T_L), \quad (4.3)$$

where  $c_{L,R}$  are the central charges of the left and right sectors, which will be assumed to be equal  $c_R = c_L = c$ .

The temperatures and entropies of the outer and inner horizons are related to the non-extremality parameter  $t_0$  by

$$2S_{\pm}T_{\pm} = r_0, \quad (4.4)$$

which implies for the temperatures of the left and right sectors

$$4S_{L,R}T_{L,R} = r_0. \quad (4.5)$$

In the extremal limit

$$S_L \rightarrow 0, \quad T_R \rightarrow 0, \quad T_{\pm} \rightarrow 0, \quad S_{\pm} \rightarrow S_R, \quad (4.6)$$

and both  $S_R$  and  $T_L$  remain finite and are convenient quantities to work with. In particular, we can express the central charge that the CFT should have in order to reproduce the Bekenstein-Hawking entropy consistently with this picture, in terms of these two parameters:

$$c = \frac{12}{\pi^2} \frac{S_R}{T_L}. \quad (4.7)$$

## Acknowledgments

The authors would like to thank Patrick Meessen for very useful conversations. This work has been supported in part by the Spanish Ministry of Science and Education grant FPA2009-07692, the Comunidad de Madrid grant HEPHACOS S2009ESP-1473 and the Spanish Consolider-Ingenio 2010 program CPAN CSD2007-00042. The work of CSS has been supported by a CSIC JAE-predoc grant JAEPRe 2010 00613. TO wishes to thank M.M. Fernández for her permanent support.

## References

- [1] S. Bertini, S. L. Cacciatori and D. Klemm, [arXiv:1106.0999 \[hep-th\]](#).
- [2] A. Strominger, C. Vafa, *Phys. Lett.* **B379** (1996) 99-104 [[arXiv:hep-th/9601029](#)].
- [3] G. Compère, [arXiv:1203.3561 \[hep-th\]](#).
- [4] A. Strominger, *JHEP* **9802** (1998) 009 [[arXiv:hep-th/9712251](#)].
- [5] M. Bañados, C. Teitelboim and J. Zanelli, *Phys. Rev. Lett.* **69** (1992) 1849 [[arXiv:hep-th/9204099](#)].
- [6] J. D. Brown and M. Henneaux, *Commun. Math. Phys.* **104** (1986) 207.
- [7] S. Carlip, *Class. Quant. Grav.* **15** (1998) 3609 [[arXiv:hep-th/9806026](#)].
- [8] M. Guica, T. Hartman, W. Song and A. Strominger, *Phys. Rev. D* **80** (2009) 124008 [[arXiv:0809.4266 \[hep-th\]](#)].
- [9] A. Castro, A. Maloney and A. Strominger, *Phys. Rev. D* **82** (2010) 024008 [[arXiv:1004.0996 \[hep-th\]](#)].
- [10] Y. Q. Wang and Y. X. Liu, *JHEP* **1008** (2010) 087 [[arXiv:1004.4661 \[hep-th\]](#)].
- [11] H. E. Camblong and C. R. Ordonez, *Phys. Rev. D* **71** (2005) 104029 [[arXiv:hep-th/0411008](#)].
- [12] S. Ferrara, G. W. Gibbons, R. Kallosh, *Nucl. Phys.* **B500** (1997) 75-93 [[arXiv:hep-th/9702103](#)].
- [13] P. Meessen and T. Ortín, *Phys. Lett.* **B707** (2012) 178 [[arXiv:1107.5454](#)].
- [14] T. R. Govindarajan, V. Suneeta and S. Vaidya, *Nucl. Phys. B* **583** (2000) 291 [[arXiv:hep-th/0002036](#)].
- [15] D. Birmingham, K. S. Gupta and S. Sen, *Phys. Lett. B* **505** (2001) 191 [[arXiv:hep-th/0102051 \[hep-th\]](#)].

- [16] K. S. Gupta and S. Sen, Phys. Lett. B **526** (2002) 121 [arXiv:hep-th/0112041].
- [17] E. Franzin and I. Smolic, JHEP **1109** (2011) 081 [arXiv:1107.2756 [hep-th]].
- [18] P. Galli, T. Ortín, J. Perz, C. S. Shahbazi, JHEP **1107** (2011) 041 [arXiv:1105.3311].
- [19] P. Meessen, T. Ortín, J. Perz and C. S. Shahbazi, arXiv:1204.0507 [hep-th].
- [20] D. A. Lowe and A. Skanata, arXiv:1112.1431 [hep-th].